Many of us in the analytic tradition continue to have a rather strange relationship with Frege. Of course, his name is shrouded in reverence; and rightfully so: he is the grandfather of analytic philosophy. But we also tend to think of the historical Frege with a sort of anachronistic puerility, and while noting the subtle peculiarities in his thought (or, more obvious ones, e.g., Russell's paradox), we excuse them as understandable consequences of being a groundbreaking thinker. After all, imagine going home tonight and developing standard quantified predicate logic ex nihilo! Frege's “two-dimensional” logical notation in particular is often considered in just this light: as an eccentric notational variant of the standard Peano-style symbolism of *Principia Mathematica*. Danielle MacBeth's new book takes exception to this reading of Frege's work. This short, yet somewhat dense book should be greeted as a delightful contribution by those who may have thought that there is something more unique and powerful about Frege's logic, over and beyond its influence on Russell, Wittgenstein and Carnap.

MacBeth's argument can perhaps be summerized by a very simple question: is Frege's logic a quantificational logic? Her answer is a qualified no—that is, it depends upon what you mean by 'quantification' in this sense. Frege's system is characterized by two divergent types of formalized generalities: those for which he reserved the Latin italic letters, or “genuine hypotheticals,” and also those which use German 'Gothic' letters which are accompanied by a concavity within the content stroke. On the received reading of Frege's logic, the former correspond to free variables while the latter denote bound variables that fall under a 'quantifier,' the concavity notation.
MacBeth's reading of Frege's work hangs upon an ambiguity in the standard logical treatment of generality, in which merely contingent generalities and lawful generalities are formalized in the same way by modern quantificational logic. However, they should be distinguished since the two types of generalities occupy different inferential roles within arguments; the former type can serve as premises from which one might reason, while the latter it would be “redundant” to do so—as both Sextus Empiricus and Lewis Carroll have noted (27-8). Inferences of the former type are truth-functional, merely formally valid, and thus are considered true in virtue of the falsity of its antecedent clause (the 'subcomponent'). The latter, in contrast, are “inferentially stronger” insofar as they support counterfactual reasoning and thus do not function as premises which ground a valid inference, but instead are materially valid and act as “inference licenses.” MacBeth argues that Frege's Begriffsschrift conditionals using Latin italic letters should be understood “not as a claim from which one reasons but instead as a principle or rule according to which one reasons.” (28, 35)

The first chapter of the book serves to explain Frege's theoretical starting point, and also to motivate her specific interpretation of Begriffsschrift conditionals. Frege's logicist programme requires that the laws of mathematics be, in principle, reducible to logic; or more properly, mathematical laws (e.g., the law of mathematical induction) are nothing more than a species of logical law. A consequence of this is that there can be no specifically 'mathematical' law—or in other words, all mathematical inferences can be expressed in terms of logical inference. Further, the Begriffsschrift conditionals differentiate themselves from the standard quantificational claims which lie at the level of fact, whereas the conditionals Frege uses are instances of laws, or “inference licenses.”

Evidence for this claim can be garnered from the particular nature of the two-
dimensional symbolism, which is the topic of her second chapter. For instance, the \textit{Begriffsschrift} sentence:

\begin{center}
\begin{tikzpicture}
  \node (A) at (0,0) {A};
  \node (B) at (0,-1) {B};
  \node (C) at (0,-2) {C};
  \draw (A) -- (B) -- (C);
\end{tikzpicture}
\end{center}

can be read as a standard conditional $C \rightarrow (B \rightarrow A)$. But the single thought can be variously analyzed, not in virtue of a set of rules of replacement to govern the equivalence, but based solely upon the character of the \textit{Begriffsschrift} conditional itself. Such conditionals as $B \rightarrow (C \rightarrow A)$ and $\neg A \rightarrow (B \rightarrow \neg C)$, which are provable equivalents in standard logic, are merely conceptual consequences of the same thought: “. . . such transitions are not acts of reason that take one from one thought to another, but only transitions from a thought in one form to that thought in another form.” (58)

MacBeth argues that Frege was not aware of the expressive power of his early logic until the early 1890's. According to her account, the alternating usage of Latin italic letters and the concavity notation with German Gothic letters is logically justified, and Frege eventually began to realize this. \textit{Begriffsschrift} inferences involving Latin italic letters serve to express second-level (or higher) expressions of concepts in the third-level relation of subordination, whereas “the same judgment expressed using the concavity and German letters has . . . the form of a subsumption of first-level concepts under a higher level concept.” (87) In this sense, the Latin italic letters can serve “to raise everything up a level.” (179) The simple case is with variables that refer to objects in a given relation, for instance, “Romeo” and “Juliet” under the second-level relation of “loving.” But, so too can the variables in the Latin italic conditional stand for relations themselves, moving the inference to higher levels, taking concepts and not objects as arguments for functions.
As Frege began to understand by 1891, the laws of logic “are at once contentful and also qualitatively different from the laws of the special sciences.” (104) They are, so to speak, laws of laws.

The conclusion we are brought to is that Frege's Begriffsschrift sentences are not merely a more clumsy version of our current “one-dimensional” Peano-style notation, as Frege's logic is not merely a logic of particulars as the Russelian logic is. It is, contrary to this reading, far more expressive and dynamic endeavor, “not a concept-script at all . . . it is a Sinnsschrift, a formula for language of thought.” (140)

An historical interpretation can often be evaluated from the perspective of its explanatory power. The advantage of such a reading is that it sheds light from a different angle than the received reading of Frege. When viewed from this new perspective, not only the distinction of Sinn from Bedeutung, but also Frege's law V from the Grundgesetze—which are the topics of her fourth and fifth chapters, respectively—can be better understood. If MacBeth is right, then Russell's paradox generated by Law V can be seen as an error on Russell's part, since the expressive tools to deal with the paradox (viz., the level-hierarchy of the theory of types) is implicit in Frege's logic. It was Frege's logicist programme that was the inspiration for the development of law V, in the development of an adequate of number; thus its dismissal should signal a problem with his logicism, and not be treated as an internal problem within his logic itself.

One of the more obvious lessons of MacBeth's book concerns the way we understand Frege historically. Historians of philosophy are often tempted (as with Frege, but also with Wittgenstein and Heidegger) to analyze a historical corpus into sharp divisions, especially when important insights have led to drastic rethinking of the early work of a given historical figure. According to such a reading there is an early Frege, the
Frege of the *Begriffsschrift*, and a 'second' Frege, armed with the powerful insight into the separation between *Sinn* and *Bedeutung* (as well as the distinctions between concept/object and function/argument), who wrote the *Grundgesetze*. If the substantive difference between the early and the later logic is the introduction of Axiom V, and if Macbeth is right that it is unnecessary, then the later logic is an anomaly, not the logic of the *Begriffsschrift*. This implies a more organic unity in the development of his thought than is typically acknowledged.

Macbeth's account of Frege also situates him in the center of several currents in contemporary analytic philosophy. The last few decades have seen a tremendous re-thinking of the task of Analytic philosophy. An important step in this process is made by re-reading, and thus re-thinking, Frege's contribution at the dawn of this philosophy of analysis. MacBeth's book in this sense can be seen as carrying forward the project set forth so prominently by Dummett, and continued by Sluga and others, to re-capture Frege as a thinker in his own right and (so to speak) to 'de-Russell' Frege. In *An Introduction to Mathematical Philosophy*, Russell quips that he thought he was “the first person who ever read [the *Begriffsschrift*].” Historians have uncritically accepted Russell's version of Frege's logic *in toto*, and consequently Frege remains as an unfortunately misread thinker, tarnished by this Russellian reading which was disseminated throughout the early twentieth century. Further, her interpretation presents Frege as a strong ally to the recent cause of viewing logic as fundamentally expressive, pragmatic, and based upon the conceptual content involved in a given inference—privileging material inference over formal inference—and thus carrying forth the interpretation of logic popularized by Sellars and Brandom in reaction to these Russellian undercurrents. For all these reasons, MacBeth presents a poignant and well-researched interpretation, which resurrects Frege
as not only the inspiration, but also as an asset to debates in contemporary analytic philosophy.

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