Formal Rules

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All page references to Paul Tomassi, Logic (Routledge, 1999)

I. Rules of Inference

Conjuncts (&)

&I: Given two true statements, one can infer that the conjunction of the two is itself a true statement (pp. 49-50)

&E: Given that a conjunctive statement is true, one can infer that both conjuncts are true (pp. 50-2)

&I

P
Q
———
P&Q

&E

P&Q
———
P
Q

Conditionals (→)

MP: Given a true conditional statement and the truth of the entire antecedent clause, one can infer the truth of the entire consequent clause. (pp. 53-5)

MT: Given a true conditional statement and the falsity of the entire consequent clause, one can infer the falsity of the entire antecedent clause. (pp. 74-7)

MP

P→Q
P
———
Q

MT

P→Q
~Q
———
~P

Disjuncts (v)

vI: Given a true statement, one can infer that statement disjuncted with any wff is itself true. (pp. 82-5)

vE: Given a true disjunctive statement, if one can infer within a subproof that each disjunct implies the same conclusion, that conclusion is itself true. (pp. 86-93)

vI

P
———
P v Q

vE

P v Q

| P
| . . .
| C
| Q
| . . .
| C

C

Subproofs

CP: If one can derive one statement from a given assumption within a subproof, one can assert that one statement implies the other. (pp. 56-66)

RAA: If one can derive a contradiction from a given assumption within a subproof, one can assert the falsity of that assumption. (pp. 101-5)

CP

| P
| . . .
| Q

$P \rightarrow Q$

RAA

| P
| . . .
| Q & ~Q

~P

II. Rules of Replacement

Metalogical Theorems

These rules are true in virtue of how the operators are defined. Thus, these rules are metalogical. (Df. ↔ is a rule similar to pp. 66-9, however, it is not necessary to differentiate between ↔ I and ↔ E). Df → does not appear in Tomassi.

Df. →

$$\frac{P \rightarrow Q}{\sim P \lor Q}$$

Df. ↔

P↔Q

(P→Q) & (Q→P)

Negations (~)

The rule of Double Negation is a primitive rule. (pp. 78-80) DeMorgan's is a derived rule that does not appear in Tomassi.

DeMorgan's (DeM)

$$\frac{\sim(P \,\&\, Q)}{\sim P \lor \sim Q} \qquad \frac{\sim(P \lor Q)}{\sim P \,\&\, \sim Q}$$

DN

$$\frac{\sim\sim P}{P} \qquad \frac{P}{\sim\sim P}$$