

Formal Rules

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All page references to Paul Tomassi, *Logic* (Routledge, 1999)

I. Rules of Inference

Conjuncts (&)

&I: Given two true statements, one can infer that the conjunction of the two is itself a true statement (pp. 49-50)

&E: Given that a conjunctive statement is true, one can infer that both conjuncts are true (pp. 50-2)

<u>&I</u>
P Q
—
P&Q

<u>&E</u>
P&Q
—
P Q

Conditionals (\rightarrow)

MP: Given a true conditional statement and the truth of the entire antecedent clause, one can infer the truth of the entire consequent clause. (pp. 53-5)

MT: Given a true conditional statement and the falsity of the entire consequent clause, one can infer the falsity of the entire antecedent clause. (pp. 74-7)

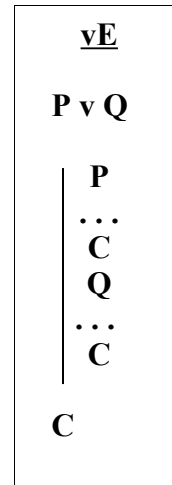
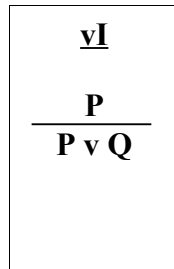
<u>MP</u>
P \rightarrow Q P
—
Q

<u>MT</u>
P \rightarrow Q \sim Q
—
\sim P

Disjuncts (\vee)

\vee I: Given a true statement, one can infer that statement disjuncted with any wff is itself true. (pp. 82-5)

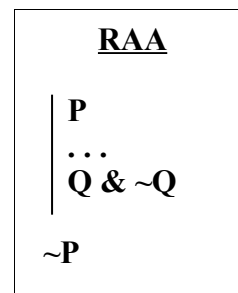
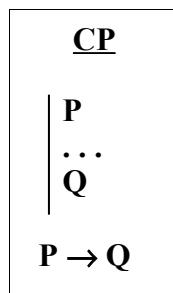
\vee E: Given a true disjunctive statement, if one can infer within a subproof that each disjunct implies the same conclusion, that conclusion is itself true. (pp. 86-93)



Subproofs

CP: If one can derive one statement from a given assumption within a subproof, one can assert that one statement implies the other. (pp. 56-66)

RAA: If one can derive a contradiction from a given assumption within a subproof, one can assert the falsity of that assumption. (pp. 101-5)



II. Rules of Replacement

Metalogical Theorems

These rules are true in virtue of how the operators are defined. Thus, these rules are metalogical. (Df. \leftrightarrow is a rule similar to pp. 66-9, however, it is not necessary to differentiate between \leftrightarrow I and \leftrightarrow E). Df \rightarrow does not appear in Tomassi.

$$\begin{array}{c} \text{Df. } \rightarrow \\ \\ \frac{P \rightarrow Q}{\sim P \vee Q} \end{array}$$

$$\begin{array}{c} \text{Df. } \leftrightarrow \\ \\ P \leftrightarrow Q \\ (P \rightarrow Q) \ \& \ (Q \rightarrow P) \end{array}$$

Negations (\sim)

The rule of Double Negation is a primitive rule. (pp. 78-80) DeMorgan's is a derived rule that does not appear in Tomassi.

$$\begin{array}{c} \text{DeMorgan's (DeM)} \\ \\ \frac{\sim(P \ \& \ Q)}{\sim P \ \vee \ \sim Q} \quad \frac{\sim(P \ \vee \ Q)}{\sim P \ \& \ \sim Q} \end{array}$$

$$\begin{array}{c} \text{DN} \\ \\ \frac{\sim\sim P}{P} \quad \frac{P}{\sim\sim P} \end{array}$$