Logic Review (Chapters 1 & 2)
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Logic is the study of **arguments**. An argument consists of three things: **premises, conclusion** and **form.** The form of an argument is how the premises serve to justify the conclusion. The form can be evaluated as either **valid, invalid,** or **sound.**

The premises and conclusions of a given argument are **statements**. Statements are evaluated as either true, false, or meaningless. Meaningless statements are not well-formed (either through ambiguity or a scope violation for a given operator). All **well-formed formula** (wff) can be evaluated as true or false.

Statements in logic consist of two types of content: **semantic content** and **logical content**. Logical content consists of the following five verbal statements and their logical equivalents:

	and	**or**	**If . . . then**	**if and only if**	**not**
symbol	&	v	→	↔	~
statement name	conjunction	disjunction	conditional implication	equivalence	negation
component name	conjuncts	disjuncts	antecedent consequent	--	--
scope	binary	binary	binary	binary	unary

Anything which is not logical content is considered semantic content and symbolized as a propositional variable (in PL).

Propositional logic is **truth functional**, meaning that the truth of the entire statement is a function of the truth values of its component propositional variables. The following chart provides the truth functions for the operators:

P	***Q***	***P & Q***	***P v Q***	***P → Q***	***P ↔ Q***	***~ P***
T	T	T	T	T	T	F
T	F	F	T	F	F	F
F	T	F	T	T	F	T
F	F	F	F	T	T	T

The **main connective** (or **major operator)** is the operator whose scope is the entire statement. Thus, the truth value of the major operator is the truth value of the entire statement.

Our system of logic is called Propositional Logic (PL). In Chs. 1 and 2 we have worked with three operators (&, →, ↔). We have four **rules of inference** and one **rule of replacement**.

Conjuncts (&)

&I: Given two true statements, one can infer that the conjunction of the two is itself a true statement (pp. 49-50)

&E: Given that a conjunctive statement is true, one can infer that both conjuncts are true (pp. 50-2)

&I

$$\begin{array}{c} P \\ Q \\ \hline P \& Q \end{array}$$

&E

$$\begin{array}{c} P \& Q \\ \hline P \\ Q \end{array}$$

Conditionals (→)

MP: Given a true conditional statement and the truth of the entire antecedent clause, one can infer the truth of the entire consequent clause. (pp. 53-5)

CP: If one can derive one statement from a given assumption within a subproof, one can assert that one statement implies the other. (pp. 56-66)

MP

$$\begin{array}{c} P \rightarrow Q \\ P \\ \hline Q \end{array}$$

CP

$$\begin{array}{l} \quad | \; P \\ \quad | \; \ldots \\ \quad | \; Q \\ P \rightarrow Q \end{array}$$

Bi-conditionals (↔)

Df. ↔: Given an equivalence between two propositional statements, one can re-phrase as a conjuct of two conditional statements, or vice versa. (This rule is similar to pp. 66-9, however, it is not necessary to differentiate between ↔ I and ↔ E)

Df. ↔

$$\begin{array}{c} P \leftrightarrow Q \\ \hline (P \rightarrow Q) \& (Q \rightarrow P) \end{array}$$

The following list of problems one should be able to do in order to demonstrate proficiency with the rules in Ch. 2:

Ex. 2.3 #2, 3, 6 – 8; Ex. 2.4 #3, 4; Ex. 2.5 #5 – 10; Ex. 2.6 #1, 2, 4; Ex. 2.7 #2, 4 – 6.